

MATHEMATICAL MODELING AND COMPUTER SIMULATION OF INSULATION SYSTEMS IN BELOW GRADE APPLICATIONS

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1. INTRODUCTION

The basic assumption for most heat loss/gain calculations included in the ASHRAE Handbook of Fundamentals is that the heat flow is one dimensional. Hence, with the exception of (radial direction) heat loss of insulated pipe or conduit, the one dimensional heat conduction equation is used in almost every case. In reality heat flow is often taking place in three directions, but the one dimensional heat conduction equation (Fick's law) is sufficiently accurate for many of these cases.

In some heat conduction problems, however, the heat flow is not one dimensional but approaches two dimensional conduction. Examples of two dimensional heat transfer include foundation walls i.e. basement, crawl space and slab-on-grade perimeter edge and metal framed walls or roofs. There is no reasonably accurate design method for solving such problems in the current ASHRAE Handbook of Fundamentals. Hence, in the past the more complex, multi-dimensional heat flow problems have not been studied because of a lack of efficient tools.

This paper introduces the results of a finite element program, which has been used to accurately simulate two dimensional heat conduction. Freezing and thawing of the ground soil can be taken into consideration with this program and time-dependent boundary conditions can be stipulated. This program was used to analyze the heat loss of a basement wall with and without insulation. Several different slab-on-grade systems were also investigated.

In the latest edition of the ASHRAE Handbook of Fundamentals, the circular heat flow pattern was assumed to be the only path of heat flow between the basement and the ground surface. A modified heat flow pattern, which included the vertical heat flow within the concrete wall, was devised. Average U values were calculated for each of these approaches and a comparison was made leading to conclusions on the relative merits of each method.

2. FINITE ELEMENT PROGRAM

In the past ten years the finite element approach has been used to analyze stresses in two dimensional problems. The same approach, however, can be used to analyze a heat conduction problem when the differential equation is converted into an integral equation.¹ The same matrix operation is used to relate all the nodes and elements with the boundary conditions at each time increment. More recently, finite element programming has been used in geothermal analyses in which the freezing and thawing of soils has been considered.

¹M. E. Gurtin, "Variational Principles for Linear Initial-Value Problems," Quart. Appl. Math. 22 (1964), 252.

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Several features of this program make it efficient in solving the two dimensional problem. It yields transient solutions, so it can accept time varying boundary conditions and the time increment size can be very large without affecting the numerical stability of the computation. Convective boundary, heat flux boundary, and heat sink or generation are considered in this program. Therefore, it can treat problems with annual cycling such as heating and cooling. It also can treat problems with hourly changes, e.g. solar radiation, etc. Materials of much different thermal properties, for example, metal and insulation, can be located next to each other without causing numerical problems in the computations.

For the basement or slab-on-grade heat loss problem, the configurations may vary slightly one from the other. It includes the surrounding earth, the foundation wall and footing, the above grade walls, the slab or basement floor and the interior space. The boundary conditions include the ambient air temperature, the indoor air temperature and the deep down ground temperature. The thermal properties of the materials used are either known or estimated. The simulation time period shall cover two heating seasons. Unless stated otherwise, the time increment used in the simulation is one week.

The air temperature does go through daily cycles. However, the temperature below the ground surface typically responds only to air temperatures averaged over several days.²

A sine-wave function which approximates the seasonal air temperature was used to generate the weekly average air temperature. The indoor temperature was assumed to range from 65 to 75°F. The deep ground temperature was chosen from the deep well data in the literature.

3. MODEL CONFIRMATION

A. One Dimensional Heat Conduction Through Multi-layered Medium. In the analysis of frost depth in a geothermal problem, Lachenbruch had developed an analytical solution to an ideal case with three layers: gravel, insulation and soil (see Table 1). The surface temperature was varied according to a sine function. Thermal properties for each of the three layers are given in Table 1.³ Lachenbruch's solution indicated that the seasonal maximum temperature at the interface of the insulation and the soil should be 32°F.

The same idealized conduction problem was solved with the finite element program, yielding a seasonal maximum temperature of 31.68°F. The result changes only slightly with the size of the time increment used as shown in Table 2.

One may conclude that in this case the result from the finite element program is independent of the time increment from 18 to 150 hours.

B. Two Dimensional Heat Conduction With Freezing and Thawing. The approximate analytical solution of A. L. London and R. A. Seban^{4,5} and their experimental results were compared to computer generated data.

²C. Sepsy, et al. Field Measurement of Below Grade Temperature at Columbus Test House. 1974-1975 and 1975-1976.

³A. Lachenbruch, "Periodic Heat Flow in a Stratified Medium with Applications to Perma-frost Problems," Geological Survey Bulletin 1083-A, U. S. Government Printing Office, Washington, DC, 1959.

⁴A. L. London and R. A. Seban, "Rate of Ice Formation," Trans. ASME, October 1943.

⁵R. A. Seban and A. L. London, "Experimental Confirmation of Predicted Water Freezing Rates," Trans. ASME, January 1945.

A metal pipe contained water at 32°F and at time-zero the pipe surface temperature was changed to -1.9°F. The ice/water interface was predicted by the analytical solution and checked against experimental data.

This cylindrical ice formation problem was also solved with the finite element program using a time increment of 1/2 minute. The approximate analytical solution, the finite element program results and the experimental data all showed close agreement (Figure 1).

C. One Dimensional Heat Conduction With Freezing and Thawing. A finite difference program was used to solve an ice propagation problem.

Initial condition: water at 32°F
Time step size for integration: one hour in both cases
Maximum time step size tried: 3 hours - finite difference
15 hours - variational method

The thermal properties used for this analysis are listed in Table 3.

The same problem was modeled with the finite element program. The results are plotted in Figure 2; agreement between the two computer simulations is good.

4. COMPUTER SIMULATIONS

A. Basement Wall. Three sets of simulations were performed. The foundation wall section considered in the first set, shown in Figure 3, consisted of an eight foot poured concrete wall (one foot above grade and seven feet below grade), 2x6 sill plate, 2x10 box sill and an R-11 stud wall with R-5.4 insulation sheathing (Figure 3).

An indoor air temperature of 70°F was assumed for the area above the basement ceiling and the basement floor temperature was 65°F. The outdoor air temperature was varied according to the sine function $T=50 + 39(\sin(\frac{\text{Days}}{365} \times 2\pi - \phi))$, approximating a total exposure of 7500 heating degree days. The convective coefficient was assumed to be 6.0 (or R-0.167) at the exterior surface and 1.47 (or R-0.68) at the interior surface. Heat loss was recorded only when the outdoor air temperature was below 65°F. The thermal properties of the different materials involved are listed in Table 4.

Seven cases were studied in the first set of simulations. The objectives were to examine partial and full height insulation systems and interior and exterior applications, while evaluating the effect of insulation in the box sill and sill plate areas. From the results listed in Table 5, the following conclusions can be drawn:

- 1) B vs C, D vs E and F vs G. The exterior application is much more efficient than the interior application for both partial insulation and complete full height insulation systems.
- 2) A vs B or D and A vs C or E. The incremental benefit of the top four (4) feet of insulation is much larger than the incremental benefit of bottom four (4) feet.
- 3) B vs F and C vs G. The impact of box sill and sill plate insulation is quite significant.
- 4) C vs D. When the box sill and sill plate are insulated, partial (top four feet) insulation on the exterior performs better than complete, full height insulation on the interior.

The foundation wall examined in the second set of simulations consisted of an eight foot concrete block wall (one foot above and seven feet below grade), 2x6 sill plate, 2x9 box sill and an R-11 stud wall with R-5.4 insulation

sheathing. The box sill is insulated with an R-11 blanket on the interior and an R-5.4 insulation board on the exterior. The sill plate is insulated only with exteriorly installed R-5.4 insulation board.

Nine cases were studied. The primary goal here was to determine the effect of convective looping within the concrete block wall. Direct and indirect evidence^{6,7} exists indicating that walls with hollow concrete blocks can transfer heat much faster than poured concrete walls. A possible convective path for concrete block foundation walls is given in Figure 4. In an effort to approximate the convective heat transfer, the conductivity of the concrete wall in vertical direction was increased from 0.75 to 2.5 BTU-ft/hr-ft²°F.

Referring to Table 6, the benefit of exterior application over interior application increases as the convection within the hollow cores becomes significant. With no convection (Cases B and C) the exterior insulation application outperforms the interior application by 25%. When increased vertical convection transfer is simulated, 33% less heat is lost (Cases H and I). The actual benefit should be more than this since the exterior application tends to cut down the driving force for convection (Cases G and C).

The construction schematic for the third set of simulations was similar to the detail for the second set except the box sill and sill plate are insulated only with R-5.4 insulation board on the outside. The purpose of this set of simulations was to determine the significance of insulating the above grade portion of the basement wall (12 inches of exposed wall). Figures 5 and 6 show that the interior insulation extends from the top of the basement wall, whereas the exterior insulation begins at the grade line, one foot below the top of the wall.

The heat loss for one heating season is tabulated in Table 7 for the five cases studied. It appears that insulating the above grade portion of the basement wall (12 inches in this case) is very important for both interior and exterior applications. If R-5.4 insulation is applied either on the inside or on the outside of the basement wall (8 feet high), basement wall heat loss can be decreased by nearly one-half compared to the uninsulated wall losses.

B. Slab-on-Grade. Four slab constructions were studied. Each one was simulated at three climatical conditions. The impact of metal stud walls and buried heating ducts on slab heat loss was evaluated. The slab constructions are illustrated in Figures 7 and 8.

The three climatical conditions are listed in Table 8a.

The thermal properties of the materials and compositions used in these analyses are given in Table 8. The heat loss of the slab edge and the slab foundation wall was computed and accumulated in the simulation. The indoor condition was maintained at 70°F and the convective coefficients were assumed to be 5.0 (R-0.2) at the exterior surface and 1.67 (R-0.67) at the interior boundary condition.

The seasonal heat loss for each case is listed in Table 9. In Case C a heating duct temperature of 110°F was assumed during the heating season. A duct temperature of 65°F was chosen for the remainder of the year. When the heating plant is active, the heating duct temperature can be as high as 150°F. The same holds true for baseboard heating systems. Hence, the 110°F assumption is reasonable.

⁶H. R. Martin, P. R. Achenbach and R. S. Dill, "Effect of Edge Insulation Upon Temperature and Condensation on Concrete-Slab Floors," MBS Building Materials and Structures Report 138, 1953.

⁷R. H. Socolow et al., "Style and Vintage as Determinants of Energy - Costly Faults in U. S. Residential Housing," Princeton, New Jersey, 1979.

In Case D the simulated heat transfer path included the metal studs and metal plates. A metal plate extended the entire length of the perimeter so the slab edge heat loss was adversely affected even in areas between the metal studs. It is likely that the real situation would be less severe than indicated. In both Cases C and D the performance of R-5.4 insulation board was quite impressive.

5. OTHER APPROXIMATIONS

There are two simple ways to approximate the heat loss through basement walls. ASHRAE Handbook of Fundamentals (1977 edition) adopted the approach of Latta and Boileau.⁸ Other research work has extended this single path approach.⁹ These two methodologies assume steady state conditions; and they can be used with heating degree day data.

A. Single Path Approach of Latta.⁸ A set of "circular" heat flow paths, shown in Figure 9, was assumed for the non-insulated basement wall. The circular paths found were reasonably accurate, as one can observe from the field measurements given in Figure 10. Basically the calculation consisted of summing the series of resistances along each path at different depths. One heat transfer value (U_i) for each depth was calculated, and the average heat transfer value (U_{av}) was obtained for the entire depth. The U_{av} value, multiplied by the product of degree days (heating) and lineal footage of basement wall, yielded the estimated heat loss through the below grade portion of the basement wall.

For insulated basement walls the same circular heat transfer paths were assumed with the resistance of the insulation added to that of each path. The U_i 's and U_{av} were calculated as before. The heat loss for both partially insulated or completely insulated basements was estimated. A table of U_i 's for insulated and non-insulated basement walls is given by Latta (Table 10).

This approach contains two major drawbacks. First, when the insulation is added to the basement wall, the heat flow paths shall be distorted and no longer circular. Secondly, it does not consider the vertical heat flow path through the basement wall. Therefore, not only may it underestimate the heat loss, but also it does not distinguish the interior and exterior applications. The second point could be critical for concrete block walls where convective looping plays a major role in the vertical heat transfer. According to the simulations by the finite element program, the exterior application can be 25 to 32% more efficient than the interior application.

B. Two Path Approach. A vertical heat flow path within the concrete wall was added, resulting in a heat flow network incorporating both parallel and series heat flow regimes. Refer to Figure 11; the heat transfer attributed to both the circular path (Path B) and the vertical path (Path A) was calculated by the following equation:

$$R_{Total} = R_C + \frac{1}{\frac{1}{R_A} + \frac{1}{R_B}}$$

where R_C is the combined resistances of the inside air film and the four inches of concrete block wall. R_A is the combined resistances of the concrete wall (vertical distance) and the outside air film. R_B is the combined resistances of four inches of concrete, the earth following the circular path, and the outside air film. This is illustrated in Figure 11 for the 6th concrete block from the top.

⁸J. K. Latta and G. G. Boileau, "Heat Loss from House Basements," Canadian Building. Vol. XIX, No. 10, October 1969, p39-42.

⁹W. Strzepek and F. S. Wang, "Below Grade Insulation Design," The Dow Chemical Company internal report.

For the insulated case, the heat flow path through the earth (Path B) was modified to reflect the distortion caused by the insulation. Refer to Figure 12; the heat flow Path B of the 6th concrete block was changed from circular to vertical straight line. This is due to the insulation on the exterior of the wall. Below the insulation the circular paths were changed to diagonal straight lines. Again, this is due to the effect of insulation above. The total resistances were calculated like in the non-insulated case.

Once the total resistance for each heat flow path is estimated, the average heat transfer coefficient (U_{av}) can be determined. The main drawback of this approach is that the vertical heat flow path was repeated for each depth increment, and no consideration was given to the mutual effect of the block wall temperatures and the vertical heat flows as a whole. This usually gives larger vertical heat loss than that of the real situation. The second drawback is that a prior knowledge of the heat flow pattern is needed to set the heat flow paths. In complicated cases, this may not be easy.

Figure 13 shows the four basement wall configurations analyzed by simple approaches and finite element program. The average heat transfer coefficients from different approaches are listed in Table 11. It appears that the single path approximation agrees reasonably well with the finite element approach. The two path approximation yields U_{av} that is too high.

6. CONCLUSIONS

The single path approach of Latta is simple to use. However, it does not differentiate interior and exterior insulation applications. In cases where the exposed portion of the concrete wall is relatively large and/or uninsulated, the estimated heat transfer coefficients may be too low.

The two path method tends to overestimate the vertical heat flow and gives a much higher heat loss or U_{av} . It should not be used without some modifications.

The finite element computer program is very useful in cases where no simple solution is available, such as slab-on-grade perimeter heat loss. In addition, other more complex issues can be addressed to, such as non-parallel heat flows in metal frame buildings.

Simulations with this method led to the following conclusions:

- 1) Exterior foundation wall insulation applications are 25 to 32% more efficient than interior applications.
- 2) Box sill and sill plate insulation have a significant impact on foundation heat loss. They may reduce the heat loss through the foundation wall by 20 to 36% (Table 7).
- 3) Insulation located near the top of the wall is more beneficial than insulation placed at the bottom of the wall.

The finite element computer program can be considered as a useful tool in solving the heat conduction problems with two dimensional geometry. However, like most other computer programs, it needs to be verified on individual cases before one can have confidence in the results. The verification process is rather difficult for field installations. Largely this is because of the vast amount of input information that can affect the heat conduction process. Solar load, local wind speed and direction, the moisture content of the soil, the identification of the soil, moisture effect on the R values of the insulation, and so on are necessary variables for an accurate simulation of a field installation.

Laboratory studies by Slusarchuk¹⁰ and Smith¹¹ indicated that compared to measured data, the finite element method gave conservative results. It also appeared from the same studies¹¹, the colder the climate the more conservative results were obtained through the finite element simulations. In spite of this drawback, the finite element program¹² should be utilized more often in earth insulation applications.

¹⁰W. A. Slusarchuk, "Frost Heave Protection for Shallow Foundations Using Artificial Perimeter Insulation," M. Sc. Thesis, University of Guelph, August 1967.

¹¹W. J. Smith, "A Study of Insulated Shallow Foundations, Timmins, Ontario, 1974-1976," M. Sc. Thesis, University of Toronto, 1976.

¹²F. S. Wang, "User's Manual for the Finite Element Heat Conduction Computer Program," June 1980, The Dow Chemical Company, Midland, Michigan 48640.

TABLE 1
THERMAL PROPERTIES OF A THREE LAYER PROBLEM

Material	Thickness	Thermal Conductivity	Volumetric Heat
Gravel	33"	0.726	22.5
Insulation	3"	0.019	0.702
Soil	33½"	1.45	31.0

$$\text{Surface Temperature} = 25.68 + 24.2 \sin \left(\frac{\text{Day}}{365} 2\pi - \phi \right) ^\circ\text{F}$$

$$\text{Ground Temperature At Great Depth} = 25.68^\circ\text{F (Permafrost)}$$

TABLE 2
ONE DIMENSIONAL HEAT CONDUCTION THROUGH MULTI-LAYERED MEDIUM. SINUSOIDAL TEMPERATURE AT THE SURFACE; BUT NO PHASE CHANGE.

Time Increment (Hours)	Max. Temp. Below Insulation °F
18.75	31.678
37.5	31.68
75.0	31.68
150.0	31.679

TABLE 3

THERMAL PROPERTIES OF ICE-WATER INTERFACE PROPAGATION

		Solid Phase	Liquid Phase
Thermal Conductivity	BTU/Hr Ft °F	1.3	0.3
Density	Lb/Ft ³	62.4	62.4
Volumetric Heat	BTU/Ft ³ - °F	31.2	62.4
Heat Of Fusion	BTU/Ft ³	8986	8986
Freezing Temperature	°F	32.	32.

TABLE 4
THERMAL PHYSICAL PROPERTIES OF MATERIAL USED*

Material	Conductivity	Heat Capacity
	$\frac{K \text{ BTU-FT}}{\text{hr-ft}^2-\text{°F}}$	$\frac{\text{BTU}}{\text{cu ft-°F}}$
2x4 stud wall, 16" oc R-11 batt plus R-5.4 sheathing, 1/2" drywall and wood/aluminum siding	0.04	2.4
Wood member	0.067	10.6
Insulation sheathing	0.016	0.61
Concrete blocks, 2 cores per block	0.64	12.75
Poured-in-place concrete	0.75	28.
**Frost susceptible soil		
Frozen	1.09	27.2
Thawed	0.66	40.0
(latent heat 3,672 BTU/cu ft)		
Gravel	0.67	21.0

* ASHRAE Handbook of Fundamentals, 1977 Edition

**Eli Robinsky, "Design of Insulation Foundations," Journal of the Soil, Mechanics and Foundations Div., September, 1973.

TABLE 5
RESULTS OF FIRST SET OF SIMULATION 7,600 D.D. HEATING

Case	Basement Wall Insulation	Sill Box Insulation	Heat Loss Per Season Per Lineal Foot In \bar{M} BTU	
			Including Sill Box & Plate	Thru Eight Foot Wall
A	None	None	0.426	0.367
B	R-5.4, Top 4 Ft. On The Interior	R-7.7 Batt Interior	0.228	0.21
C	R-5.4, Top 4 Ft. On The Exterior	R-5.4 Exterior	0.183	0.162
D	R-5.4, Full Height Interior	R-7.7 Batt Interior	0.193	0.175
E	R-5.4, Full Height Exterior	R-5.4 Exterior	0.143	0.122
F	Same As B-1	None	0.274	0.22
G	Same As B-2	None	0.28	0.15

TABLE 6
RESULTS OF SECOND SET OF SIMULATIONS 7,600 D.D. HEATING

<u>Case</u>	<u>Basement Wall Insulation</u>	<u>Concrete Wall Conductivity</u>	<u>Heat Loss Per Season Per Lineal Foot In \bar{M} BTU</u>
A	None	K-0.75	0.384
B	R-5.4, Top 4' Interior	K-0.75	0.231
C	R-5.4, Top 4' Exterior	K-0.75	0.173
D	Same As A	K-1.5	0.483
E	Same As B	K-1.5	0.273
F	Same As C	K-1.5	0.19
G	Same As A	K-2.5	0.537
H	Same As B	K-2.5	0.297
I	Same As C	K-2.5	0.2

TABLE 7
RESULTS OF THE THIRD SET OF SIMULATIONS*

<u>Case</u>	<u>Basement Wall Insulation</u>		<u>Heat Loss Per Season Per Lineal Foot Of Parimeter In \bar{M} BTU</u>
	<u>Below Grade Portion (7 Ft.)</u>	<u>Above Grade 1 Foot</u>	
A	None	None	0.376
B	R-5.4 On Interior	R-5.4 On Interior	0.19
C	R-5.4 On Interior Top 3 Feet	R-5.4 On Interior	0.24
D	R-5.4 On Exterior Top 4 Feet	None	0.31
E	R-5.4 On Exterior 7 Feet	None	0.28

*7,600 D.D. Heating

TABLE 8a
CLIMATICAL CONDITIONS

Location	Annual Mean Air Temperature	Seasonal Variation	Degree Day Heating Per Season
Madison, Wisconsin	46.1	27.5	7,433
Columbus, Ohio	52.5	22.9	5,350
Atlanta, Georgia	60.8	18.3	2,950

TABLE 8
THERMAL PROPERTIES OF CONSTRUCTION MATERIALS*

Material	Thermal Conductivity BTU-ft/hr-ft ² -°F	Heat Capacity BTU/cu ft-°F
Poured concrete	0.75	28.
Sand/gravel fill	0.67	21.
Insulation board	0.016	0.61
**Frost susceptible soil		
Frozen	1.09	27.2
Thawed	0.66	40.0
(latent heat 3,672 BTU/cu ft)		
Brick	0.75	40.0
Concrete block 8"	0.64	12.75
Hollow core 4"	0.46	12.75
***Air space behind brick veneer	0.14	0.018
Steel stud	26.2	59.
Stucco	0.42	28.
Drywall	0.093	20.
Wood trim	0.068	15.

* ASHRAE Handbook of Fundamentals, 1977 Edition

** Eli Robinsky, "Design of Insulation Foundations," Journal of the Soil, Mechanics and Foundations Div., September, 1973.

***Natural convection assumed

TABLE 9
SEASONAL HEAT LOSS FOR DIFFERENT CONSTRUCTIONS AND CLIMATIC CONDITIONS

Slab-On-Grade Constructions		Madison Wisconsin 7,433 D.D. M BTU	Columbus Ohio 5,350 D.D. M BTU	Atlanta Georgia 2,950 D.D. M BTU
Case-A	Insulated	87.	64.	40.
	Non-Insulated	111.	87.5	51.
Case-B	Insulated	85.	63.	38.6
	Non-Insulated	144.	107.	65.6
Case-C	Insulated	114.	92.2	63.4
	Non-Insulated	328.	272.	193.
Case-D	Insulated	91.	67.6	41.4
	Non-Insulated	205.	154.	94.6

TABLE 10
HEAT LOSS BELOW GRADE: Btu/(hr) (*F) (ft²)
Insulation with k = 0.24 Btu/(hr) (ft²) (*F/in.)
= 0.02 Btu/(hr) (ft) (*F)
Soil k = 0.8 Btu/(hr) (ft) (*F)

Depth (ft)	Path Length through Soil (ft)	-----Heat Loss-----			
		Uninsulated	1-in. Insulation	2-in. Insulation	3-in. Insulation
0-1 (1st)	0.68	0.410	0.152	0.093	0.067
1-2 (2nd)	2.27	0.222	0.116	0.079	0.059
2-3 (3rd)	3.88	0.155	0.094	0.068	0.053
3-4 (4th)	5.52	0.119	0.079	0.060	0.048
4-5 (5th)	7.05	0.096	0.069	0.053	0.044
5-6 (6th)	8.65	0.079	0.060	0.048	0.040
6-7 (7th)	10.28	0.069	0.054	0.044	0.037

TABLE 11
AVERAGE HEAT TRANSFER COEFFICIENTS FROM DIFFERENT APPROACHES

Uav BTU/Hr-°F-Ft. Of Wall	Base Case	Case 1	Case 2	Case 3
Single Path Approx.	1.82	1.23	0.82	0.7
Two Path Approx.	2.6	2.32	1.35	1.23
Finite Element	2.08	1.6	0.96	0.72

CYLINDRICAL FREEZING

$r_o = 0.466''$

$r =$ Ice-Water Interface

$t_o = -1.9^\circ\text{F}$

$t_f = 32.0^\circ\text{F}$

$h_o = 21.6 \text{ BTU/Hr-Ft}^2\text{-}^\circ\text{F}$

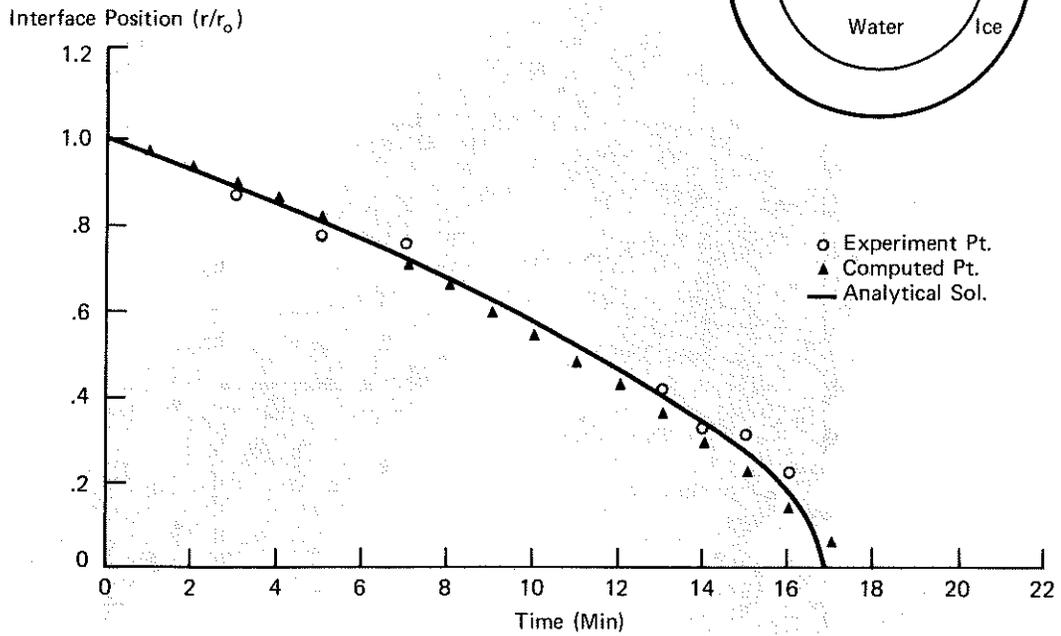
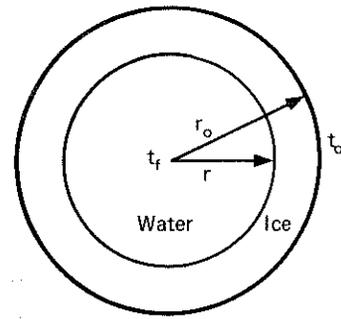


Fig. 1 Two dimensional heat conduction

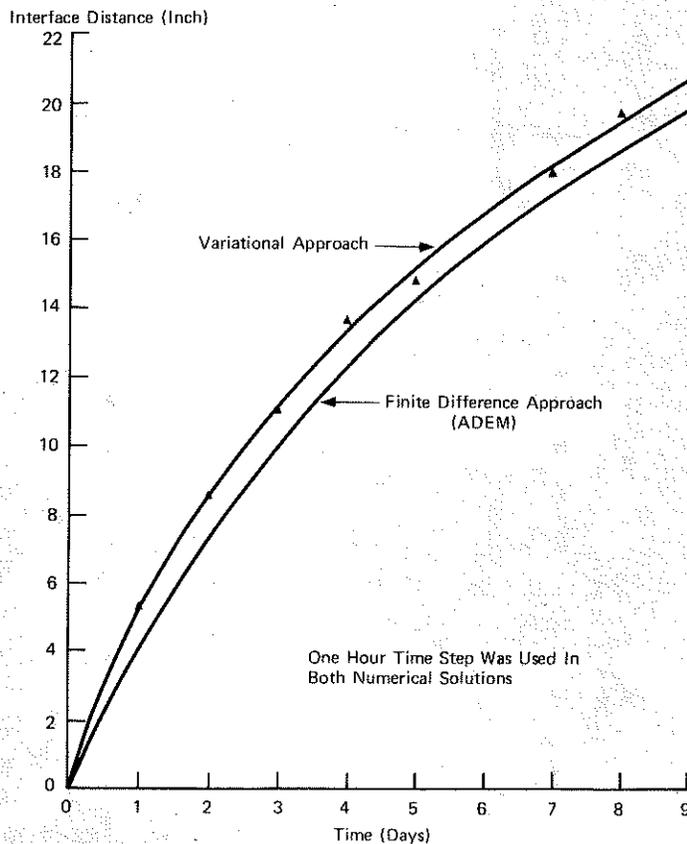


Fig. 2 One dimensional heat conduction with phase change freezing of water

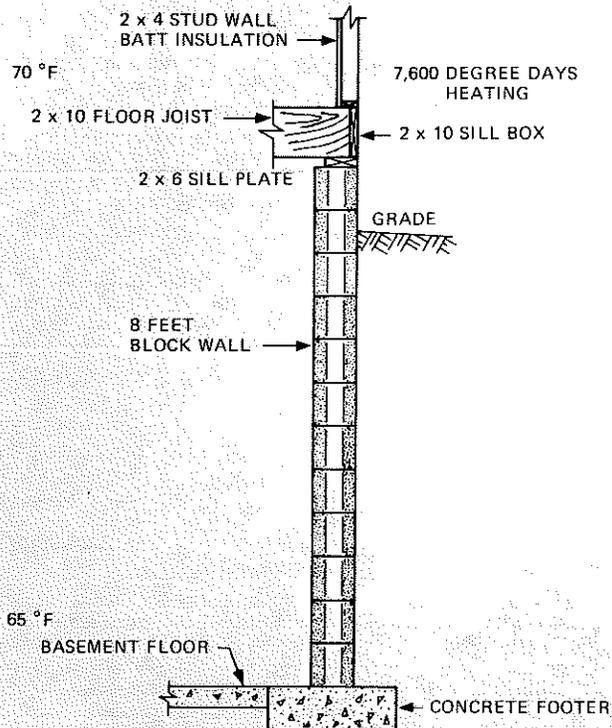


Fig. 3 Common configuration of basement wall

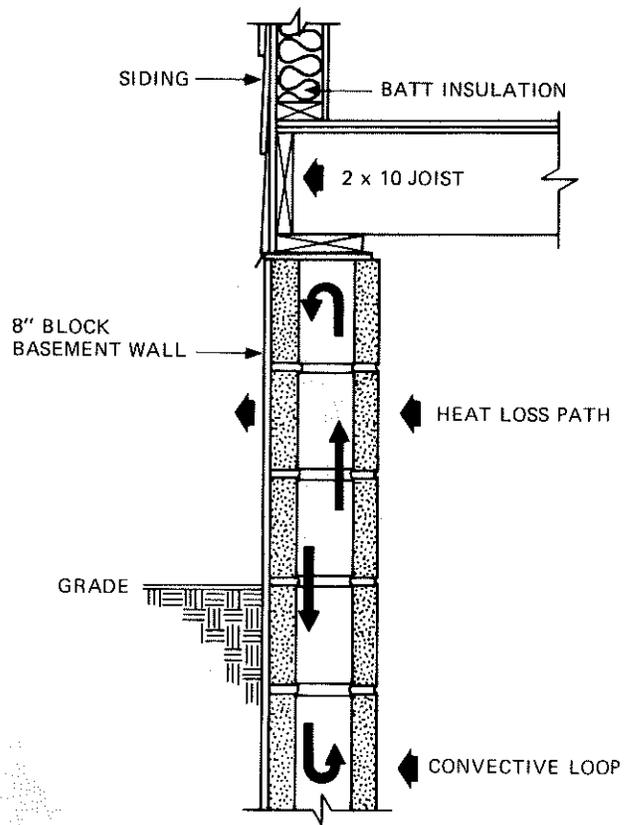


Fig. 4 Possible convective path

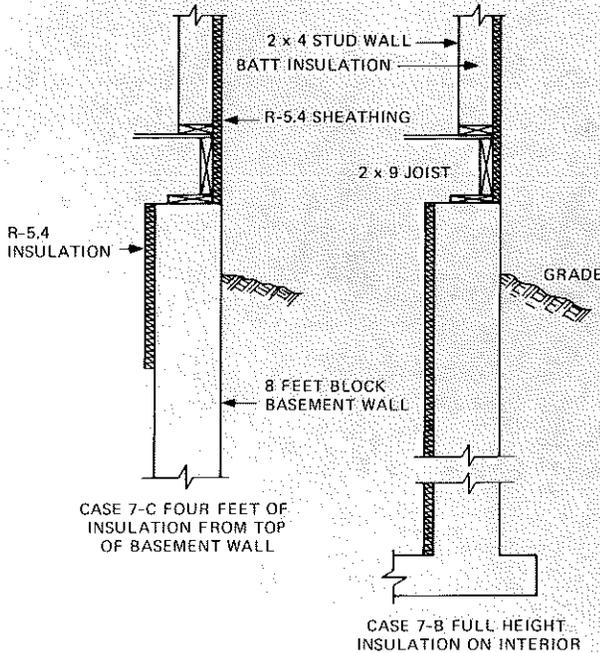


Fig. 5 Interior application of insulation

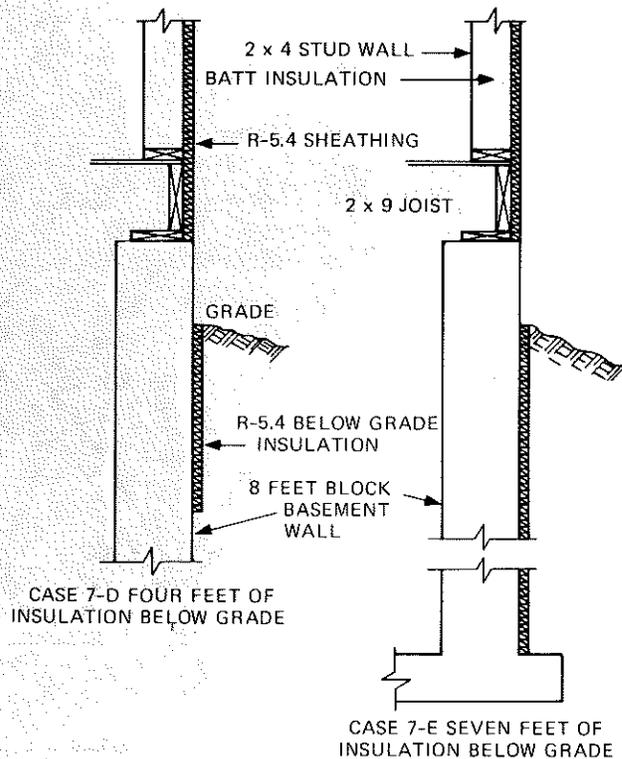


Fig. 6 Exterior application of insulation

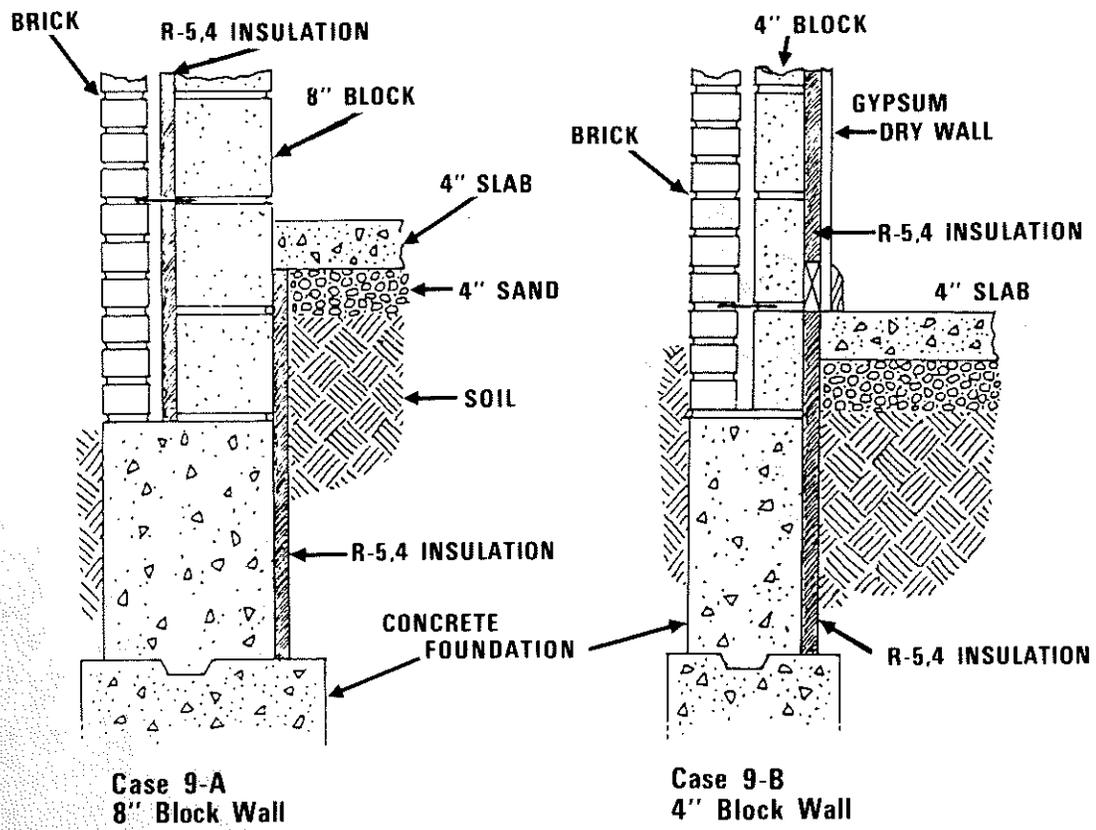


Fig. 7 Slab on grade foundation insulation

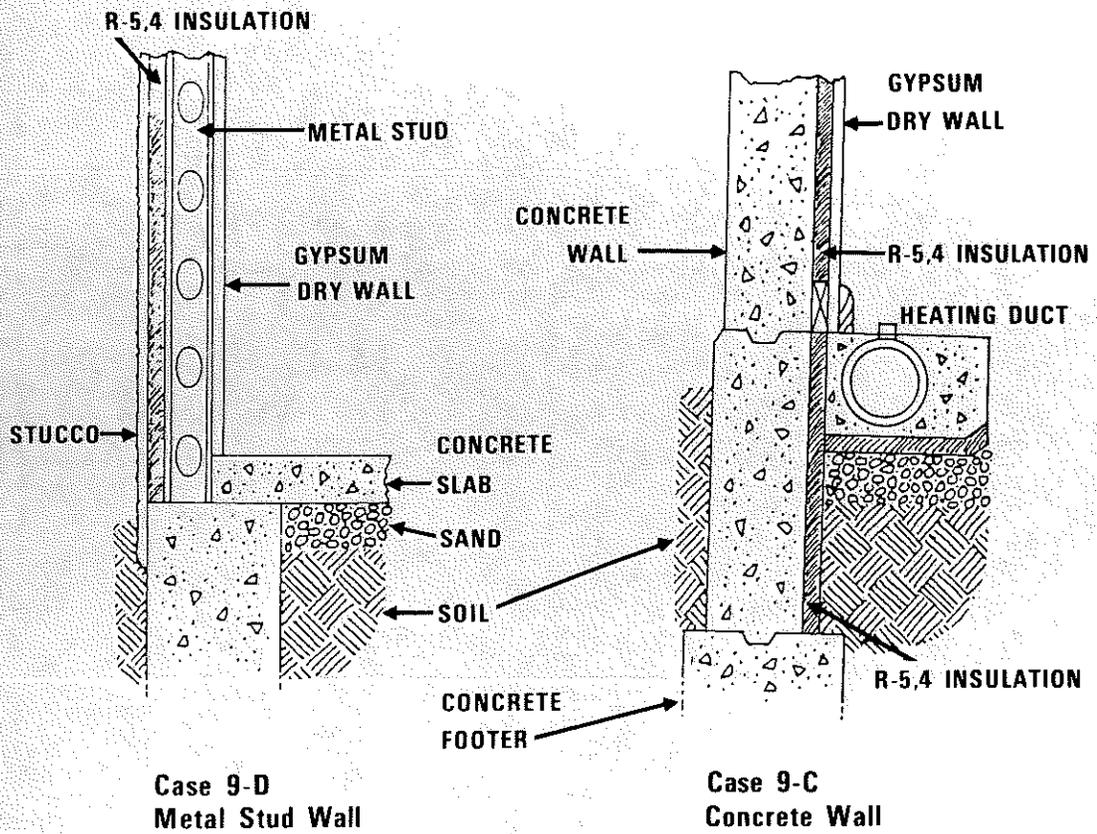


Fig. 8 Slab on grade foundation insulation

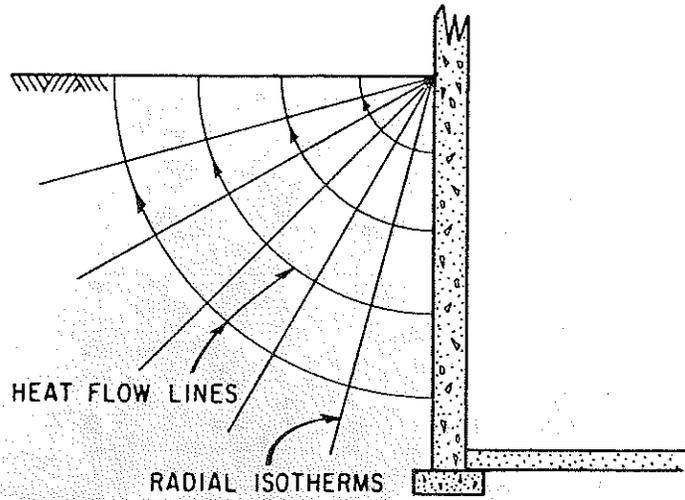


Fig. 9 Paths of heat flow from a basement wall to the ground surface

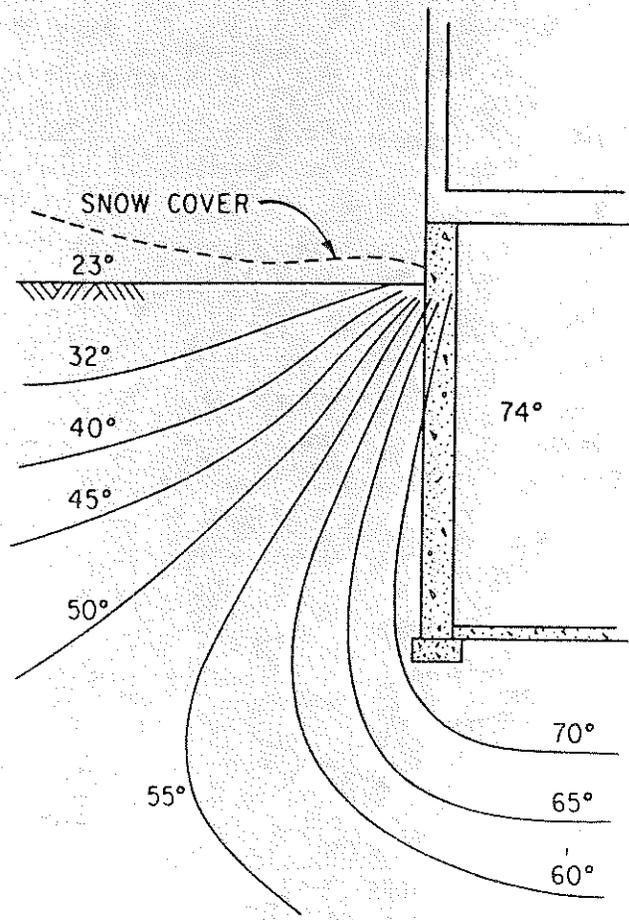


Fig. 10 Isotherms of ground temperatures measured at Saskatoon

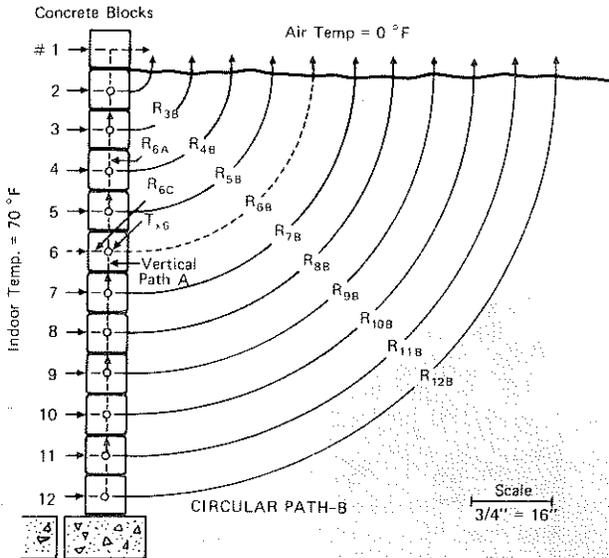


Fig. 11 Heat flow path A & B un-insulated basement wall

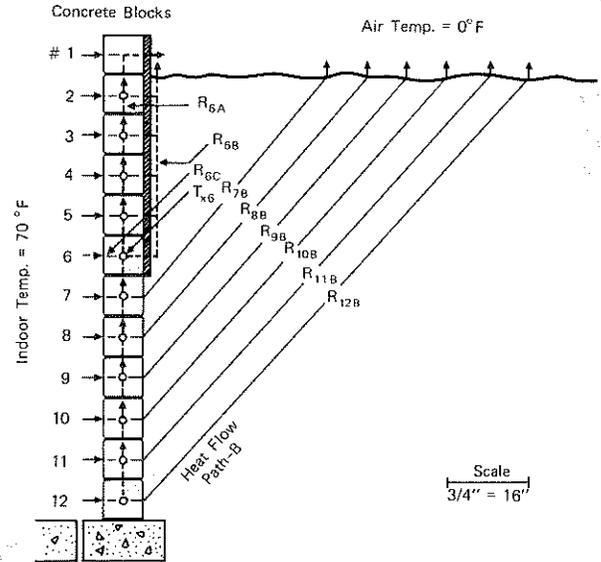
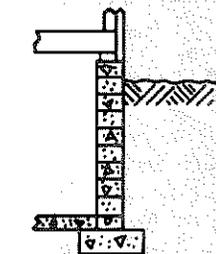
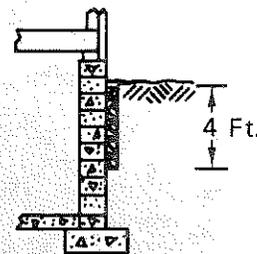


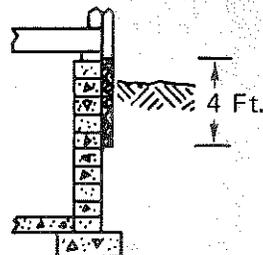
Fig. 12 Heat flow path A & B for partially insulated basement wall



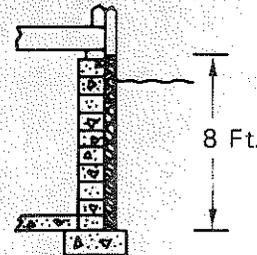
BASE CASE UN-INSULATED



CASE 1 FOUR FEET INSULATION (R-5.4) BELOW GRADE



CASE 2 FOUR FEET INSULATION (R-5.4) FROM THE TOP



CASE 3 FULL HEIGHT INSULATION (R-5.4)

Fig. 13 Four basement wall insulation designs analyzed by simple approaches and finite element program